

Limits on the neutron–antineutron oscillation time from the stability of nuclei

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Abstract

We refute a recent claim by Nazaruk that the limits placed on the free–space neutron–antineutron oscillation time $\tau_{n\bar{n}}$ can be improved by many orders of magnitude with respect to the estimate $\tau_{n\bar{n}} > 2(T_0/\Gamma)^{1/2}$, where T_0 is a measured limit on the annihilation lifetime of a nucleus and $\Gamma \sim 100$ MeV is a typical antineutron–nucleus annihilation width.

In a recent letter [1], Nazaruk claims to have increased the limit obtained from the stability of nuclei on the free–space $n \rightarrow \bar{n}$ oscillation time [2–4] by 31 orders of magnitude. In view of the startling nature of this claim, it is important to carefully inspect the derivation of this result. In this note, we point out a specific error in Nazaruk’s derivation. Correcting it, one obtains a limit of the same order of magnitude as given by previous authors [2–4] who used potential models for the n –nuclear and \bar{n} –nuclear dynamics. We also explain in very simple terms the origin of the standard limit.

Nazaruk’s framework is that of the S matrix in the diagrammatic approach. For neutron–antineutron oscillations in the nucleus, he writes down (Eq. (17) of Ref. [1]) the probability $W(t)$ for the transition at time t as

$$W(t) = \epsilon^2 t^2 - \epsilon^2 \int_0^t dt_\alpha \int_0^{t_\alpha} dt_\beta W_{\bar{n}}(t_\alpha, t_\beta), \quad (1)$$

where $\epsilon = \tau_{n\bar{n}}^{-1}$ characterizes the free–space oscillation, and $W_{\bar{n}}$ is given by Eq. (12) of Ref. [1] (after noting that $W_{\bar{n}} = -2iT_{ii}^{\bar{n}}$ by comparing Eqs. (16) and (17)):

$$W_{\bar{n}}(t_\alpha, t_\beta) = -2 \sum_{k=1}^{\infty} (-i)^k \left\langle \int_{t_\beta}^{t_\alpha} dt_1 \dots \int_{t_\beta}^{t_{k-1}} dt_k H(t_1) \dots H(t_k) \right\rangle. \quad (2)$$

We will follow Ref. [1] in demonstrating the result for the oversimplified case where the \bar{n} –nuclear dynamics is reduced to just a width factor, $H = -i\Gamma/2$, with $\Gamma \sim 100$ MeV being a typical value. The introduction of real potentials for the n and \bar{n} does not change the order of magnitude of the result [3, 4]. The r.h.s. of Eq. (2) is easily evaluated, resulting in

$$W_{\bar{n}}(t_\alpha, t_\beta) = 2(1 - \exp[-\Gamma(t_\alpha - t_\beta)/2]), \quad (3)$$

which replaces Nazaruk’s Eq. (18)

$$W_{\bar{n}}(t_\alpha, t_\beta) = 1 - \exp[-\Gamma(t_\alpha - t_\beta)]. \quad (4)$$

Substituting Eq. (3) in Eq. (1), one obtains

$$W(t) = \frac{4\epsilon^2}{\Gamma} t [1 - 2(1 - \exp(-\Gamma t/2))/\Gamma t], \quad (5)$$

instead of Nazaruk's erroneous result (Eq. (19) of Ref. [1])

$$W(t) = \epsilon^2 t^2 \left[\frac{1}{2} + \frac{1}{\Gamma t} - \frac{1}{\Gamma^2 t^2} (1 - \exp(-\Gamma t)) \right]. \quad (6)$$

The time dependence of $W(t)$, Eq. (5), becomes linear in t for $t \gg \Gamma^{-1}$, that is for times considerably exceeding the very short time $\Gamma^{-1} \sim 10^{-23}$ sec characteristic of antineutron annihilation:

$$W(t) \xrightarrow{\Gamma t \gg 1} \frac{4\epsilon^2}{\Gamma} t = \frac{4}{\Gamma \tau_{n\bar{n}}^2} t. \quad (7)$$

Thus the rate of $n \rightarrow \bar{n}$ oscillations in the nucleus is given by the coefficient of t in Eq. (7). The nuclear lifetime T_0 for annihilation due to $n \rightarrow \bar{n}$ oscillations is given by the inverse of this rate:

$$T_0 = \frac{1}{4} \frac{\Gamma}{\epsilon} \tau_{n\bar{n}} = \frac{\Gamma}{4} \tau_{n\bar{n}}^2, \quad (8)$$

where the first equality shows explicitly the enhancement factor Γ/ϵ which makes the nuclear lifetime T_0 longer by a huge factor with respect to the free-space lifetime $\tau_{n\bar{n}}$. For a measured limit T_0 on the stability of nuclei, a limit on $\tau_{n\bar{n}}$ emerges:

$$\tau_{n\bar{n}} > 2(T_0/\Gamma)^{1/2}. \quad (9)$$

This is the order-of-magnitude estimate, for $\Gamma \sim 100$ MeV, derived in previous calculations [2-4].

We remark that Nazaruk's expression for $W(t)$, Eq. (6), behaves quadratically in t for $t \gg \Gamma^{-1}$. This erroneous behavior is due to the overall factor 2 missing in Eq. (4), compared to Eq. (3), so that only one-half of the free-space $\epsilon^2 t^2$ component is cancelled out in Eq. (6). The full cancellation of the $\epsilon^2 t^2$ factor is equivalent to the statement that disconnected diagrams, such as Fig. 2a of Ref. [1], cannot contribute to the S matrix and must therefore be cancelled by other diagrams (Fig. 2b of Ref. [1]). Once the $\epsilon^2 t^2$ term of Eq. (6) is dropped, the rest of the terms reproduce Eq. (5) upon replacing Γ by $\Gamma/2$ according to Eq. (3).

Ref. [1] also makes strong statements about the inability of (optical) potential models to produce a correct description of $n \rightarrow \bar{n}$ oscillations in nuclei. In fact, once the erroneous result [1] given by Eq. (6) is replaced by the correct Eq. (5), the potential model reproduces it exactly. From Eq. (22) of Ref. [1], we have

$$W_{\text{pot}}(t) = 2\text{Im}i(\epsilon/\delta U)^2 [1 - i\delta U t - \exp(-i\delta U t)]. \quad (10)$$

Substituting $\delta U = -i\Gamma/2$, the potential model expression $W_{\text{pot}}(t)$ is seen to yield precisely the $W(t)$ of Eq. (5), obtained via the diagrammatic method.

Finally, for clarity, we would like to present a simple derivation of the $n \rightarrow \bar{n}$ oscillation time in the optical model. Following Eqs. (21) of Ref. [1], we write the

time-dependent coupled Schrödinger equations for a zero momentum neutron and antineutron in external (nuclear) potentials $U_n = 0$ and $U_{\bar{n}} = -i\Gamma/2$, respectively:

$$i\frac{\partial}{\partial t}\psi_n = \epsilon\psi_{\bar{n}} \quad , \quad (i\frac{\partial}{\partial t} + i\frac{\Gamma}{2})\psi_{\bar{n}} = \epsilon\psi_n. \quad (11)$$

Operating on the first equation with $(i\frac{\partial}{\partial t} + i\frac{\Gamma}{2})$ and using the second equation to eliminate $\psi_{\bar{n}}$, we obtain

$$(\frac{\partial^2}{\partial t^2} + \frac{\Gamma}{2}\frac{\partial}{\partial t} + \epsilon^2)\psi_n = 0. \quad (12)$$

Looking for exponential decay solutions of the form $\psi_n = \exp(-\gamma t/2)$, where γ is the neutron rate of disappearance, one gets a quadratic equation for γ

$$\gamma^2 - \Gamma\gamma + 4\epsilon^2 = 0, \quad (13)$$

with solutions, to leading order in $(\epsilon/\Gamma)^2$, given by

$$\gamma_{<} \simeq \frac{4\epsilon^2}{\Gamma} \quad , \quad \gamma_{>} \simeq \Gamma. \quad (14)$$

The solution $\gamma_{>}$ should be discarded since it corresponds to the rate of disappearance of an antineutron when its oscillation coupling ϵ to the neutron is neglected. (Note that $\psi_{\bar{n}}$ also satisfies Eq. (12), the difference being in the boundary conditions.) The rate $\gamma_{<}$ agrees precisely with that obtained in Eq. (7), and hence leads to the $n \rightarrow \bar{n}$ nuclear lifetime expression of Eq. (8).

In conclusion, potential models produce the right temporal evolution of $n \rightarrow \bar{n}$ disappearance with the correct order of magnitude estimate for neutron-antineutron oscillation lifetimes in nuclei. These models [3, 4], to various degrees, account also for the single-particle n -nucleus and \bar{n} -nucleus dynamics. An important observation [3] is that most of the contribution to $n \rightarrow \bar{n}$ oscillation and subsequent \bar{n} annihilation in finite nuclei comes from the outer part of the neutron wave function. The inclusion of more sophisticated dynamical effects, such as short range two-body correlations or medium corrections to the basic process, is unlikely to change the order of magnitude of the optical potential results given by Eqs. (8), (9).

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